# HOSSAM GHANEM

## (21) 3.* Tangent Line; Vertical Tangent Line And Corner(A)

**Example 1** 37 July 12, 2003 A

Find an equation for tangent line to the graph of $f(x) = (x+1)^2 \cos x$ at $x = 0$

### Solution

$f(x) = (x+1)^2 \cos x$

$f(0) = \cos 0 = 1 \qquad P(0,1)$

$f^\backslash(x) = 2(x+1)\cos x - (x+1)^2 \sin x$

$m = f^\backslash(0) = 2(1)\cos 0 - 0 = 2$

$\therefore \quad m = 2 \qquad f(0,1)$

$y - y_1 = m(x - x_1)$

$y - 1 = 2x$

$y - 2x - 1 = 0$

**Example 2** 38 March 31, 2004

(a) Use the definition of derivative to find $f^\backslash(1)$ where $f(x) = \dfrac{1}{1+\sqrt{x}}$

(b) Find an equation of the line normal to the graph of $f$ at $x = 1$

### Solution

$$f(x) = \frac{1}{1+\sqrt{x}}$$

$$f(1) = \frac{1}{1+\sqrt{1}} = \frac{1}{2}$$

$$f^\backslash(a) = \lim_{h\to a} \frac{f(x) - f(a)}{x - a}$$

$$f^\backslash(1) = \lim_{x\to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1} \frac{\frac{1}{1+\sqrt{x}} - \frac{1}{2}}{x - 1} = \lim_{x\to 1} \frac{2(1+\sqrt{x})\left(\frac{1}{1+\sqrt{x}} - \frac{1}{2}\right)}{2(1+\sqrt{x})(x-1)}$$

$$= \lim_{x\to 1} \frac{2 - (1+\sqrt{x})}{2(1+\sqrt{x})(x-1)} = \lim_{x\to 1} \frac{1-\sqrt{x}}{2(1+\sqrt{x})(x-1)} = \lim_{x\to 1} \frac{-(\sqrt{x}-1)}{2(1+\sqrt{x})(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{-1}{2(1+\sqrt{x})^2} = \frac{-1}{2(2)^2} = \frac{-1}{8}$$

$(b)$

$$f(1) = \frac{1}{1+1} = \frac{1}{2}$$

$$\therefore \quad m = \frac{-1}{8} \qquad p\left(1, \frac{1}{2}\right)$$

$y - y_1 = m(x - x_1)$

$$y - \frac{1}{2} = \frac{-1}{8}(x - 1)$$

$8y - 4 = -(x - 1)$

$$8y + x - 5 = 0$$

## Example 3

34 March 23,2002

Find an equation of the line normal to the graph of $y = x^2 + \dfrac{x^2 - 7}{2x + 1}$ at $x = 1$

## Solution

$$y = x^2 + \frac{x^2 - 7}{2x + 1}$$

$$y^{\backslash} = 2x + \frac{(2x + 1)(2x) - (x^2 - 7)(2)}{(2x + 1)^2}$$

at $x = 1$

$$y = 1 + \frac{-6}{3} = 1 - 2 = -1$$

$$y^{\backslash} = 2 + \frac{(3)(2) - (-6)(2)}{9} = \frac{5 + 12}{9} = \frac{17}{9}$$

$$\therefore \ m = -\frac{17}{9} \qquad p(1, -1)$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = \frac{-9}{17}(x - 1)$$

## Example 4

54 November 16, 2009 A

Let $f(x) = \dfrac{x^{\frac{1}{3}}}{x - 2}$ . Find the $x$- coordinate(s) of the point(s) on the graph of $f$ at which

(a) the tangent line is horizontal (b) the tangent line is vertical

## Solution

$$f(x) = \frac{x^{\frac{1}{3}}}{x - 2} \qquad D_f = R / \{2\}$$

$$f^{\backslash}(x) = \frac{(x - 2)\left(\frac{1}{3}x^{\frac{-2}{3}}\right) - x^{\frac{1}{3}}}{(x - 2)^2} = \frac{x - 2 - 3x}{3x^{\frac{2}{3}}(x - 2)^2} = \frac{-2 - 2x}{3x^{\frac{2}{3}}(x - 2)^2}$$

$H.T$ if

$$-2 - 2x = 0 \quad \rightarrow \quad x = 1$$

$V.T$ if

$$3x^{\frac{2}{3}} = 0 \quad \rightarrow \quad x = 0$$

| Example 5<br>36 April 19, 2003 A | Show that $f(x) = \dfrac{x^3 - x + 1}{x^2 + 1}$ has a horizontal tangent in [ 0 , 1 ] |
|---|---|

Solution

$$f(x) = \frac{x^3 - x + 1}{x^2 + 1}$$

$$f'(x) = \frac{(x^2+1)(3x^2-1) - (x^3 - x + 1)(2x)}{(x^2+1)^2}$$

$$f'(0) = \frac{(1)(-1) - (1)(0)}{1} = \frac{-1}{1} = -1 < 0$$

$$f'(1) = \frac{(2)(2) - (1)(2)}{4} = \frac{4-2}{4} = \frac{1}{2} > 0$$

$f'(x)$ cont on $[0,1]$

$\therefore \exists\, c \in (0,1)$ such that $f'(c) = 0$ I.V.T

$\therefore f$ has a horizontal tangent in $[0,1]$

## Example 6

33 October 25,2001 A

Find the points on the graph of $f$ at which the tangent line is perpendicular to the line $7y + x = 1$ where $f(x) = x^3 + x^2 + 6x - 5$

Solution

$L$ : $7y + x = 1$ $\quad m = \frac{-1}{7}$

$f$ :

$$f(x) = x^3 + x^2 + 6x - 5$$

$$f'(x) = 3x^2 + 2x + 6$$

$$\therefore 3x^2 + 2x + 6 = 7$$

$$3x^2 + 2x - 1 = 0$$

$$(3x-1)(x+1) = 0$$

$x = \frac{1}{3}$ or $x = -1$

the point s $\left(\frac{1}{3}, f\left(\frac{1}{3}\right)\right)$ , $(-1, f(-1))$

## Example 7

40 August 7, 2011

(4 Points ) Find the $x-$ coordinates of the points on the graph $y = 3x^{\frac{4}{3}} + 3x^{\frac{1}{3}} + 1$ where

(a) the tangent line is horizontal (b) the tangent line is vertical

Solution

$$y = 3x^{\frac{4}{3}} + 3x^{\frac{1}{3}} + 1$$

$$y' = 4x^{\frac{1}{3}} + x^{\frac{-2}{3}}$$

$$y' = x^{\frac{-2}{3}}(4x+1)$$

$$y' = \frac{(4x+1)}{x^{\frac{2}{3}}}$$

$H.T$ if $4x + 1 = 0 \quad \rightarrow \quad x = -\frac{1}{4}$

$V.T$ if $x^{\frac{2}{3}} = 0 \quad \rightarrow \quad x = 0$

# Homework

| | |
|---|---|
| 1 | Find the equation of the tangent line to the curve $y = x(2-x)^2$ at point ( 2 , 0) |
| 2 | 49 July 5, 2008<br>Find an equation of the tangent line to the curve $y = \frac{\sqrt{x}-1}{\sqrt{x}+1}$ at $x = 1$ |
| 3 | 2 November 9, 1989<br>Let $f(x) = x^2 + 3x + 4$ Find an equation for the tangent line to the graph of $f$ at $x = 1$ |
| 4 | Find the equation of the tangent line to the curve $y = \frac{5x+7}{x+1}$ , $x \neq -1$<br>at point that are perpendicular to the line $2x - y - 7 = 0$ |
| 5 | 12 November 2, 1995<br>Find an equation of the normal line to the graph of the function $f(x) = \frac{x \sec x}{x+1} + 2$<br>at $x = 0$ |
| 6 | 46 July 5, 2007<br>Find the equation of the normal line to the curve of<br>$y = x^2 + \sec x \tan x$ at $x = 0$ |
| 7 | 51 November 24, 2008<br>Find an equation of the normal line to the graph of $f(x) = \frac{x^2}{x+3}$ , at $x = 3$ |
| 8 | Find the value of $x$ at tangent line is horizontal where $f(x) = 3x^2 - 2x + 1$ |
| 9 | 42 March 29, 2006<br>Find the $x$ – coordinates of the points on the graph of $f(x) = \frac{x+1}{2x^2 - x + 1}$<br>at which the tangent line is horizontal |
| 10 | 40 October 28, 2004 A<br>Find the point on the graph of $f(x) = \frac{x^2}{x+1}$ at which the tangent line is horizontal |
| 11 | Suppose $g(x) = x^4 - x^3 + 3x - 11$<br>Show that the graph of $g$ has a horizontal tangent line |

# Homework

| | |
|---|---|
| 12 | Let $f(x) = x^{\frac{4}{3}} - x^{\frac{1}{3}} - 2$<br>Find the points on the graph of $f$ at which the tangent line is vertical |
| 13 | 35 October 31, 2002 A<br>Let $f(x) = x^{\frac{3}{5}} - x + 7$<br>Find the points on the graph of $f$ has a vertical tangent line |
| 14 | 34 March 23,2002<br>Let $f(x) = x^{\frac{7}{3}} - 7x^{\frac{1}{3}} + 2$ Find the points on the graph of $f$ at which<br>(a) the tangent line is horizontal (b) the tangent line is vertical |
| 15 | 42 May 5, 2008<br>Show that $f(x) = (x+1)^{\frac{5}{4}} + x^{\frac{1}{3}}$ has a vertical tangent line |
| 16 | 55 April 8, 2010<br>(3pts) At what point(s) on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $6x - 2y + 7 = 0$ ? |
| 17 | 56 July 10, 2010<br>Let $f(x) = \dfrac{x^{\frac{3}{5}}}{(2x+1)}$ . Find the $x$- coordinate of the points on the graph of $f$ at which :<br>a) the tangent line is horizontal<br>b) the tangent line is vertical (4 points) |
| 18 | 57 November 8, 2010<br>Find the $x$- coordinate of the points at which $f(x) = x^{\frac{2}{3}} - x^{\frac{5}{3}}$ has<br>a) a vertical tangent line,<br>b) a horizontal tangent line. (4 pts.) |
| 19 | 14 March 28, 1996<br>Find the $x$ -coordinates of the points on the graph of $y = x + \sin^2 x - \cos x$ at which the tangent line is parallel to the line $y - x + 2 = 0$ and $x$ is in the interval $(0, 2\pi)$ |

19 — 14 March 28, 1996

Find the $x$ -coordinates of the points on the graph of $y = x + \sin^2 x - \cos x$ at which the tangent line is parallel to the line $y - x + 2 = 0$ and $x$ is in the interval $(0, 2\pi)$

## Solution

$L$ : $y - x + 2 = 0$ $m = 1$

$f$ :

$y = x + \sin^2 x - \cos x$

$\frac{dy}{dx} = 1 + 2\sin x\cos x + \sin x$

$\therefore$ $1 + 2\sin x\cos x + \sin x = 1$

$2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\text{Sin}\, x = 0$ or $\cos x = \frac{-1}{2}$

$\text{Sin}\, x = 0$ $\rightarrow$ $x = 0$ or $x = \pi$

$\cos x = \frac{-1}{2}$ $\rightarrow$ $\alpha = \frac{\pi}{3}$ $\rightarrow$ $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$